2. FUNDAMENTAL IDEAS AND OBJECTIVES OF MATHEMATICAL EDUCATION[1]

Objectives

The objectives of the school fall into two strands. The first is the narrow but popular aim of enabling the child to have a job or earn a living when he leaves school. On this there is general agreement by the state, the family, the neighbours, the public and sometimes even the child himself. But this aim should not be interpreted too narrowly. We do not know sufficiently well in advance what job is the most suitable for the child to undertake or what job he will in fact get. The school should therefore aim at developing the child widely so as to present him with many opportunities of employment from which he can make his choice. The need for a general preparation leads to the second and wider objectives of the school, which are to develop all the inner resources of the child; that is, to give him culture.

Mathematical education similarly has its narrow and wider objectives. The narrow objectives are that the school should equip the child with sufficient mathematical education which will make him employable. Even the more lowly forms of employment make some demand on mathematical education. The more modest aim of primary school mathematical education is

(1) arithmetic of natural numbers (+, -, x, ÷),

(2) length, area, volume,

(3) fractions, percentage

There is no doubt that the primary school should teach arithmetic. For all practical purposes, it is sufficient that children do their arithmetic <u>mechanically</u>, having regard for accuracy and some speed. It is desirable, however, that the children should also learn to do their arithmetic <u>insightfully</u> so that they get the results faster and obtain more permanent results.

The narrow objectives of mathematical education at the secondary school level are that the school should assist in the professional preparation of prospective technical personnel such as technicians, technologists, engineers, surveyors, scientists, managers. Mathematics above the primary school level is strictly mandatory for certain kinds of technical personnel and immediately necessary only for prospective users of mathematics. For the same reason that it cannot be decided far ahead what kind of employment will be most suitable for the secondary school leaver or what employment he will actually get, every child entering the secondary school ought to receive mathematical education which is aimed at general competence for a variety of employments.

These narrow objectives are desirable but they are not enough. Of course, they could actually lead to the wider objectives and higher ideals, namely, to

(1) serve the individual and the community

(2) develop all the inner resources of the child.

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[1] Adapted from the lead paper by Professor G. Polya.

Several of such general objectives of education have been proposed, from Plato downward, in various formulations. Here are a few:

(1) General culture

(2) Discipline of the mind

(3) Desirable habits of thinking

(4) Mental and emotional maturity

(5) A well balanced personality.

Each aim mentioned embodies some high ideal worth striving for. Yet every one of the aims may be worthless if only lip service is paid to it and it may be worse than worthless if it is used as a cheap and empty slogan. On the other hand, each of those ideals is worth while provided that one honestly believes in it and it is really valuable if one earnestly tries to live up to it and translate it into everyday practice of the school. In planning the curriculum or preparing a lesson or choosing a problem for homework, therefore, it is desirable to ask oneself "What is it good for?" If it does not promote any objective, narrow or wider, it is not worth undertaking.

Fundamental Ideas

Here are two simple rules on the process of learning. The first is that learning should be active. As expressed in a picturesque metaphor by Socrates: the ideas should be born in the student's mind and the teacher should act only as midwife. The teacher must "let the students discover by themselves as much as feasible under the given circumstances." The second rule is that there are consecutive phases in the process of learning. Things should come before words, the concrete before the abstract, doing and seeing before verbal expression and so on. In down-to-earth language, learning begins with action and perception, proceeds from thence to words and concepts and should end in desirable mental habits.

There is general agreement that mere possession of information is of little value in mathematics. To know mathematics means to be able to do mathematics, use mathematical language, find the unknown, check a proof and so on. Therefore, to teach mathematics, the student must be given the opportunity to do mathematics. There are many ways of doing mathematics but solving problems appears as the most cardinal mathematical activity. Hence it is well justified that all mathematical textbooks contain problems. The problems may even be regarded as the most essential contents of a textbook and problem-solving by the students as the most essential part of mathematical instruction.

Problems may be routine or non-routine. A routine problem has a specific aim. It should teach the student to use correctly this or that particular rule, procedure or definition. It offers drill and practice but does not demand any invention or originality. To solve a quadratic equation with given numerical coefficients is a routine problem for a student who has been shown the general formula. On the contrary, a non-routine problem challenges the student's inventiveness and originality and should aim at some more general and higher objective.

Mathematical problems should be used to implant in the minds of the students whatever attitudes and procedures may be generally useful for solving any kind of problem. This is what Polya calls the tactics of problem-solving.

In teaching problem-solving in the mathematics class, one quite naturally comes across attitudes and patterns of thought, the usefulness of which is not restricted to mathematical problems. For example, in solving a problem the right attitude is first of all to <u>understand</u> the problem, distinguish its principal parts and see each of them as clearly as one can and then try to foresee the result. Above all, one should try to <u>conceive a plan</u> before diving into details. What is described here is an attitude whose usefulness extends far beyond elementary mathematical problems. This attitude belongs to the tactics of problem-solving. Here is one procedure, mentioned in many of the older textbooks of geometry. We begin by taking for granted the result, the conclusion that we have to prove or the figure that we have to construct. Then we derive from it some other conclusion or figure, hence still another one, and so on, until we arrive at the hypothesis or the data proposed. This procedure of "working backwards", familar in problems of construction in geometry is not restricted to geometry, but has a wider interest[1]; it belongs in fact to the tactics of problem-solving.

Some of the students will, others will not, use mathematics after leaving the secondary school. For users of mathematics the tactics of problem-solving may be the beginning of their professional attitude. For non-users it may easily be the most useful thing that remains with them from the mathematics of the secondary school.

In conclusion, before giving a lesson, the teacher should ask himself "What is it good for? Does it promote any narrow or wider objective?" Unless it promotes some objective, especially the wider objectives and the higher ideals, it is not worth giving.

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1 For further discussion, see G. Polya: How to Solve It, New York, Doubleday & Co., 1957, pp. 225-232.

Selected Reading List

The views of G. Polya are further developed and illustrated in his books. Other authors express the same or similar ideas in different words. Some pursue the two broad strands of objectives along ramifying channels. A teacher should read these objectives and the fundamental ideas, not only at the beginning of a programme but from time to time, so as to maintain a clear view of the objectives and grasp the ideas and the approach which lead to successful mathematical education.

The list is by no means exhaustive as is evident from the bibliographies in some of the books.

1. Biggs, E.E. and Maclean, J.R.: Freedom to Learn, Ontario, Addison-Wesley (Canada) Ltd., 1969.

2. Commonwealth Secretariat: Mathematics in Commonwealth Schools, London, Commonwealth Secretariat, 1969.

3. Dienes, Z.P.: Building Up Mathematics, London, Hutchinson Educational Ltd., 1964.

4. Hardy, G.H.: A Mathematician's Apology, Cambridge, Cambridge University Press, 1940.

5. Land. F.W. (ed.): New Approaches to Mathematics Teaching, London, Macmillan, 1966.

6. Peel, E.A.: The Pupil's Thinking, London, Oldbourne, 1967.

7. Polya, G.: How to Solve It, New York, Doubleday, 1957.

8. Polya, G.: Mathematics and Plausible Reasoning (2 vols.), Princeton, Princeton University Press, 1954.

9. Polya, G.: Mathematical Discovery (2 vols.), London, John Wiley and Sons, 1962/65.

10. Skemp, R.R.: Understanding Mathematics, London, University of London Press, 1964.

11. Wertheimer, M.: Productive Thinking, New York, Harper, 1945.

12. Whitehead, A.N.: The Aims of Education, London, Ernest Benn Ltd., 1962.