3. MATHEMATICS IN PRIMARY SCHOOLS[1]

The teaching of mathematics in the primary school by the new approach, otherwise known as "discovery" learning, is a challenging and rewarding experience for children and their teacher. Primary school children, whose ages in most countries range from five to twelve years, learn to discover for themselves mathematical concepts, patterns and relationships and to solve problems in mathematics. In countries like Britain, where syllabuses are not externally imposed and head teachers are free to devise their own schemes of work and classroom methods, it is easy for teachers to try the new approach, which consists of getting children to work in small groups or individually, encouraging them to think for themselves and investigate mathematical problems, which they do in individual and sometimes highly original ways.

The aims of discovery learning in mathematics are

(i) to set children free to think for themselves

(ii) to give them opportunities to discover the order and pattern, which is the essence of mathematics

(iii) to give them the skills.

To promote discovery learning, the teacher should know the environment and be able to select and structure his programme. He should plan the learning of each child so that the child could discover all the mathematics and mathematical skills he should learn. The teacher should provide the right experience and ask the questions which would encourage the child to discover his own methods in written calculation as well as in making discoveries with shapes and patterns.

Discovery learning is a method that can and should be used in every aspect of mathematics, at every stage and with children of all abilities. Differences in abilities should be recognised. Some children grasp abstract ideas easily and after only a brief acquaintance with real materials; others require a lengthy period with real materials before they are able to abstract a single idea or concept. Some respond to open-ended questions, others need directed questions. It is the teacher's responsibility to provide the right experience for the child and ask him suitable questions that would enable him to take the final step and make the discovery himself. When a teacher finds himself telling the answer, he should admit that either the child is not ready for the experience or the teacher is not asking the right questions,

Here are two examples of questions, one directed and the other open-ended, in Miss Biggs' own words:

> For several years I had given considerable direction to children and teachers in isolating the variables when working with a pendulum. For example, one assignment was: "Time the pendulum for 30 swings for lengths 6", 12", 18" as far as 48". Draw a graph. Can you find from your graph the length of a pendulum which beats seconds?" A group of ten year olds performed the experiment carefully and obtained a reasonable result. A month later I discussed the pendulum with the same group. "What did you discover?",

---

[1]Adapted from the lead paper by Miss E.E. Biggs, H.M.I.

I asked. Their recollections were so hazy and confused that we we had to start at the beginning once more.

Contrast this experience with the next, in which the question was open-ended:

> Last summer I was working with a group of nine and ten year old boys in down-town New York. We had decided to time various objects which the boys had chosen to roll down a long slope in the corridor. We had no stop watch, so I had added a length of fine string and a piece of plasticine to our collection. When the boys asked for a stop watch, I asked them if they could devise a means of timing from the materials I had provided. On seeing these Richard immediately suggested making a pendulum. It so happened that there was a large hook fixed at a height of 7 feet above the slope, to which the boys attached the longest pendulum they could make. "Where shall we start?" I asked. "Up at the ceiling, straight out", Earl replied. "Does it matter where we start?" I questioned. They decided that it did matter and we set the pendulum swinging. "Does it beat regularly?" I asked. "Let's count," they replied. But the boys decided that the pendulum was swinging too slowly for effective timing. "How shall we change the beat?", I asked. "Shorten the string," suggested Richard. "Lengthen the string," said Adrian. "Add more plasticine," said Robert. They experimented with different lengths until they were satisfied with the beat. "How shall we count?" I asked. "Number of swings in half a minute," said Mervyn. I gave him my watch with a second hand and he was soon timing confidently from any starting point of the second hand. "It swings much faster when we shorten the string," they commented as they continued their experiments with different lengths, "but it doesn't change when we alter the weight." These experiments were rough and ready and needed a careful follow-up with a good point of suspension, but I want to stress that <u>all</u> the thinking had been done by the boys themselves and the suggestions came from them and not from me. "The sense of personal discovery influenced the intensity of their experience and vividness of their memory."[1] We were so engrossed that we forgot all about the purpose of the timing![2]

The initial impulse needs to arise from some experience in which the children are interested. For example, the directed question to find the length and breadth of a room may seem to be of little interest compared with the open-ended question "Do you think this room is twice as long as it is wide? In how many ways can you find out without actually measuring?" which Miss Biggs reported to have aroused the enthusiasm of some nine-year olds.

Discovery learning could be achieved in arithmetic. Here are some topics which will be of interest to children and which with the right questions could give children a sense of achievement and of discovery:

1H.M.S.O.: Children and their Primary Schools, London, H.M.S.O. 1967, para. 549.

2Commonwealth Secretariat: Mathematics in Commonwealth Schools, London, Commonwealth Secretariat, 1969, p.38.

(i) Matching sets (one-one correspondence).

(ii) Counting, matching a number name to each object.

(iii) Cardinal numbers, numbers in sequence, the number line.

(iv) Measuring; length, weight, capacity, time, area, using simple instruments.

(v) Operations of addition, subtraction, multiplication and division with number, length, weight, capacity, money, time.

(vi) Number relationships and the basic laws - commutative, associative and distributive - preceding written calculations. Observation and discovery of patterns and relationships should come before memorisation. For efficient calculation, some memorisation is desirable but this should be reduced to a mimimum.

(vii) Following the knowledge of number and number relationships, children should be given the experience which would encourage them to discover their own methods for long multiplication and division.[1]

Mathematical symbols should be introduced only <u>after</u> varied experience. The steps should be

(i) active experience by the child.

(ii) discussion with other children and with the teacher.

(iii) recording in the child's own words.

(iv) introduction of mathematical symbols.

(v) written calculations devised by the child himself and refined with guidance from the teacher.

(vi) practice as necessary.

In considering mathematical content in the primary school, the school environment and the teacher's mathematical background are two important aspects. There are topics that arise in the classroom and the school premises and from the children's experience in the home, the shops, the streets and the farms according to the environment. Examples are

(i) statistics, collecting data, and expressing them in words and by graphs, reading and interpreting them.

(ii) shapes, common three-dimensional bodies such as cubes, cuboids, cylinders, spheres, prisms, pyramids, polyhedra, plane surfaces such as squares, rectangles, polygons, triangles, circles, etc. and the use of graph paper.

---

[1]For a view on long multiplication and division, see p. 16.

(iii) symmetry, in nature as seen in the environment (fruits, leaves, flowers, animals), man-made as illustrated in patterns of various kinds and in many materials including paper, cloth, wood, etc.

(iv) similarity, of solid and plane figures.

(v) limits, small and large, leading to some idea of infinity.

(vi) transformation of translation, rotation, reflection, enlargement.

When children begin to organise their basic number facts and later their knowledge of fractions, the introduction of structural material[1] and devices like the number line help the children to summarise and gain further insight into their experiences. However, children should first have extensive experience of the environment, handle and deal with numbers in practical situations before using commercial structural material.

1See A.T.M.: Mathematics Teaching, No. 24, A.T.M., Autumn 1963.

Suggestions for Action

1. Mathematics in the primary school should be seen as a new approach in both method and content. The teacher should therefore familiarise himself with both aspects through in-service courses and by working through one or more teachers' and source books in primary school mathematics. He should himself do the kind of mathematics to which his children would be exposed.

2. The teacher should acquire practice in planning his lesson, supplying the necessary materials and providing suitable experience and then asking the right questions in the directed or open-ended form as the situation demands.

3. In communities where most children have no more than primary education, the teacher should anticipate the need for social mathematics in adult life. This implies doing more arithmetic topics but the method should consistently lead the children to find out for themselves.

4. The new approach favours group work and individualised teaching. The teacher should therefore keep in view the immediate and future needs of the children who go on to a secondary school or who have to take a selection examination for admission to a secondary school.

5. The teacher and the children should cultivate an eye for mathematics in their environment and the habit of collecting materials for use in mathematics lessons. Here is a list of useful materials, which can be extended or modified according to the environment:

(a) Sorting, matching and counting materials, structural materials:

Seeds, stones, shells, small plastic toys, cubes, rods (length 1 to 10 units), number track, a number line pinned on the wall or marked on the ground in a variety of relevant units, beads on a taut horizontal string.

(b) Shapes:

Boxes, round tins and other containers; leaves, shells, fruits, seeds, flowers, inch and centimetre cubes, beads, balls, globe, mirrors.

(c) Measurement:

Length; canes or softwood, string or fibre, homemade trundle wheel (yard or metre in circumference). Weight; balance scales, springs, coiled wire (hair rollers, extension springs, tendrils of climbing plants). Lever; straight stick or rod to suspend, or to balance on a small wedge of wood, metal washers. Capacity: pots, gourds, coconut shells, pails, tins. Time; string and bob for pendulum. Area; various materials to cover a plane (or curved) surface or a model, e.g. textiles, newsprint, leaves, seeds, squares, identical triangles, etc., mats, geo-boards or peg boards made of softwood with pins or pegs. Rotation; compass, geared wheels, protractor, home-made clinometer.

(d) Constructional Materials:

Commercially produced strips and bolts or drinking straws, building blocks of various shapes and sizes (off-cuts), home made level.

(e) Materials for Communication and Recording:

Squared paper (in inch and centimetre units), coloured paper and card, newsprint or other cheap paper, paints or dyes, brushes, coloured pencils or felt-tipped pens, abacus and rings of dough or paper beads, scissors.

6. A balance should be kept between practical work and abstractions. Many children enjoy practical activities and mathematical games and others are fascinated by number patterns and visual representations in the form of tables, charts and graphs. It is desirable that activities lead to the abstraction of mathematical ideas. Here are some constructional and other activities which give varied opportunities for investigation:

(a) building structures with unit cubes and unit cuboids.

(b) making patterns with unit squares, unit rectangles, unit triangles etc.

(c) making frameworks with split bamboo canes, pawpaw trunks, drinking straws or meccano, making cages from palm piths, baskets and coops from palm leaves.

(d) model-making in card and other materials: sheds and buildings as well as the regular solids (tetrahedron, cube, etc.).

(e) Interesting shapes such as bandstands, drums, balls, ice cream cones, the scoop for serving ice cream; pyramids could be made in the classroom and their properties investigated.

(f) making or investigating toys, for example kites, tops and hoops, doll-dressing. Mechanical toys and devices are not easily available in all countries, but some could be home-made, for example model aeroplanes, wheeled toys and an inclined plane (for experiment), go-carts, slings, geo-boards (for discovering relations of areas of different shapes).

(g) Games such as cricket, football, basket ball and tennis can make children aware of the path of a moving object.

(h) Paper folding, experiments with paint and mirrors lead to investigations of symmetry and enlargement (similarity). Children sometimes invent "coordinates" for themselves when they describe the position of buried treasure on an imaginary island they have drawn. When trying to find the treasure they realise the value of order in the pairs of distances used to fix the position of the treasure and ordered number pairs are then accepted. Coordinates can also be used in symmetrical reflections and other transformations.

The following are the respective mathematical ideas which can be derived from the preceding activities.

(a) The properties of various two-dimensional shapes, from making and handling frameworks with 3, 4, 5, etc. sides, using first equal then unequal strips; the rigidity of the triangle. Sequences of numbers from a series of frameworks of regular polygons made rigid by longer struts.

(b) Ideas of angles from a child's own observations of the hands of a clock and of other changes of direction. Angle properties of parallel lines and angle sum of a triangle from patterns made in paper folding. Rotation and gear ratio from cog wheels of different sizes.

(c) Mathematical similarity (enlargement of scale) in three dimensions, by building a sequence of cubes from unit cubes. Paths on the globe and on the earth. Similarity in two dimensions by building a sequence of squares from unit squares (and other shapes). Maps. Recognition that some shapes (e.g. cubes, spheres, squares), are always similar.

(d) Volume of cubes, spheres, cylinders, cones and pyramids from practical experience, for example, comparison of the weights of clay used to make the objects, or of the water contained by or displaced by them.

(e) Area: approach through irregular shapes, for example comparing the area of two leaves; counting the number of yam heaps covering each of two fields. Many activities lead to the understanding and application of area, such as gardening, weaving mats, doll dressing, making pictures with scraps of fabric. The making of nets of solids gives a useful link between two and three dimensions.

(f) The speeds of various moving objects compared with the child's own speeds.

(g) Transformation: the making of decorative patterns leads to an awareness of the various movements of a unit shape required to make different types of pattern: translation along a line, rotation about a point, reflection. Correspondences of points, and relations between lines and angles, can be investigated. The basic idea of congruence can be firmly established by means of these experiences.

7. There are a number of pupils' books in modern mathematics. The teacher should select one of the series of such books, usually one of those which were written for the locality. Where there is no local series, the choice should be from those which require little adaptation. It is desirable, however, to have a local series which is based on the children's environment, experience and interest.

8. When the medium of instruction is a mother tongue which does not contain words for expressing certain mathematical experiences and concepts, extra care is needed in finding suitable words and expressions in the mother tongue for describing mathematical experiences and concepts. The teacher should identify and recognise the difficulties which arise from cultural differences with a view to overcoming the difficulties. In many such countries, efforts are being made to find appropriate words and expressions in the mother tongue.

9. In countries where the schools cannot afford commercial structural materials such as Stern, Dienes', Cuisenaire, Avon, Structa, Unifix,[1] which are apparatuses from which to make mathematical abstractions, the teacher should devise or improvise aids which are made with local materials. What is important is that such aids and the environment should provide sufficient experience for the children in learning mathematics.

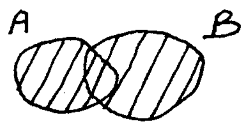
10. The algebra of sets is a common feature in many primary school mathematics books and it makes a lively start, leading on to sorting out, matching, one-one correspondence, numbers, operations, symbols, co-ordinates, graphs. The teacher should be thoroughly conversant with the language, visual representation by Venn diagrams and the symbols, which should be introduced after the children have grasped the concepts. The more usual symbols are:

$\in$	belongs to or is a member of the set
$\subset$	is contained in
$\exists$	there exists
$\cup$	union, sometimes read 'cup'
$\cap$	intersection, sometimes read 'cap'
$\{a, b, c\}$	the set of the elements a, b, c.
$\mathcal{U}$	the universal set or the universe
$\emptyset$	the empty set
A'	the complement of the set A
$\rightarrow$	tends to
$>$	greater than

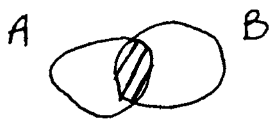
[1]These apparatuses are described in detail respectively in Mathematics Teaching Nos. 16, 17, 18, 19, 20 and 21.

$<$	less than
$=$	equal to
$\simeq$	approximately equal to or is equivalent to the set.

<u>Venn Diagrams</u> - Examples



shaded part is $A \cup B$



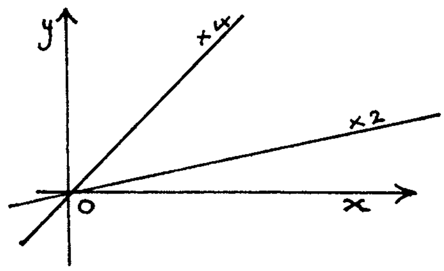
shaded part is $A \cap B$

11. Graphs are a fascinating topic with many applications in number patterns and in daily life. Examples should include simple statistical representations by pictographs, pie graphs, block graphs, column graphs, line graphs.

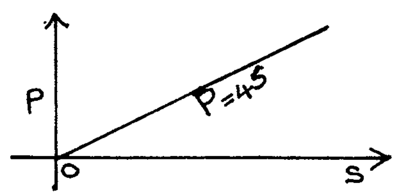
Certain number patterns may be represented by continuous line graphs:

(a) Straight lines

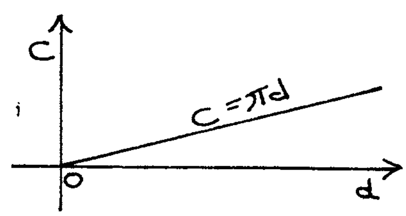
(i) The pattern of the graphs of multiplication tables of 2, 3, 4, etc. as straight lines of increasing steepness. Note the extension to multiplication tables of -2, -3, -4.



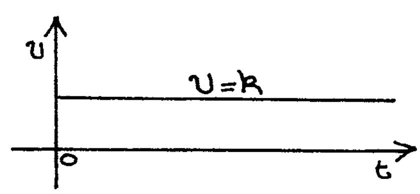
(ii) The perimeter/side graph for squares.



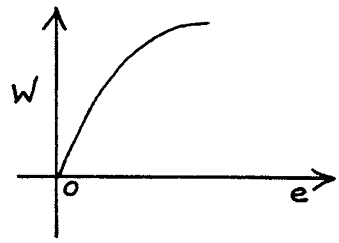
(iii) The circumference/diameter graph for circles.



(iv) A graph showing constant speed..



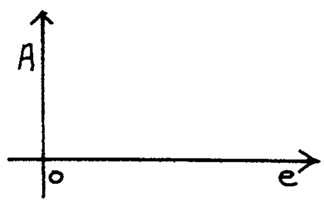
(v) The extension of a spring as weights are added to one end, provided a certain limit of weight is not exceeded.



Children sometimes predict that the graph of the multiplication tables or perimeters of squares will be a straight line because of the equal differences.

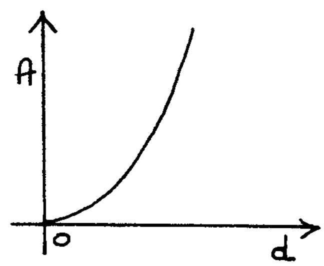
Edge of Squares	1	2	3	4	5
Perimeter	4	8	12	16	20
Differences		4	4	4	4

(b) Graphs of squares and cubes



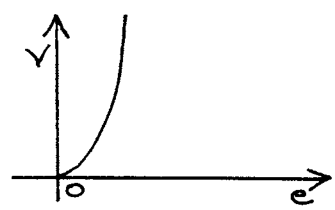
(i) The area/edge graph for the squares.

(ii) The area/diameter (or radius) relation for circles found approximately by counting squares. The number pattern of the area of square is soon recognised.



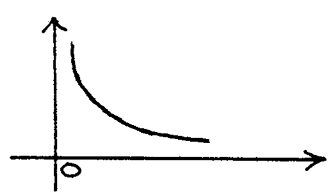
Edge of Squares	0	1	2	3	4	5
Area of Squares	0	1	4	9	16	25
Differences		1	3	5	7	9

Some children realise that because these differences are not equal, the graph will not be a straight line. Because the differences increase and form the odd number pattern they expect a rising curve.



(iii) The volume of cubes and of spheres will be investigated and will lead in a similar way to a curve.

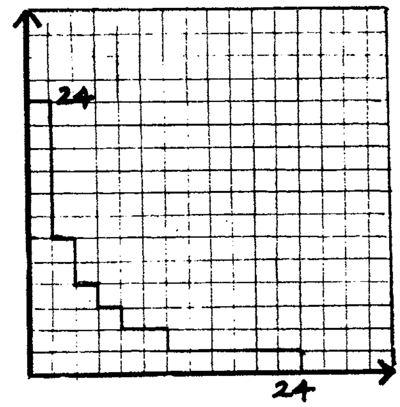
(c) Constant product curve



(i) Examples of the constant product pairs have already arisen, e.g. in the multiplication square.

(ii) Dimensions of rectangles made with 24 squares. The complete set of rectangles with integral dimensions can be cut out and put in order: 1 by 24, 2 by 12, 3 by 8, 24 by 1. When children are asked to arrange these in order taking up the least possible space, they sometimes overlap them and then recognise the similarity to the pattern in the multiplication square.

Ordered pairs, (1,24), (2,12), ..., (24,1), with relation lb = 24 give a continuous curve.



12. As a primary school teacher usually handles all the subjects in his class, he should make use of the links of mathematics with other subjects of the curriculum, especially with science, social studies, arts and crafts. A class may be given a project which demands the knowledge of mathematics, science and social studies. In the rural areas, agricultural situations should form the basis of the problems. Interesting mathematical problems could arise from poultry farms, fishponds, pig, cocoa, citrus, and rice farms, sale of produce, banana, coconut and oil palm farm practices.

13. The teacher should be involved in the making of the syllabus for his class. He should select those topics which arouse the interest of the children, assist them in their exploration and discovery and enrich their mathematical experience. The degree of computational skill required will depend on the society and its environment. Where calculating machines, ready reckoners and the slide rule are in common use, long multiplication and division skill is no longer necessary. Here is a suggested curriculum from which a syllabus could be made. The teacher should feel free to re-arrange the topics in the best way dictated by his experience of the children and their problems and language and in relation to the other subjects of the school curriculum.

1. Mathematical concepts of number and space drawn from concrete familiar objects within the experience of the children.

2. Sets and sub-sets, leading to union, intersection and complements of sets and sub-sets.

3. Numerals in the mother tongue and English.

4. Symbols, the denary number system and the digits.

5. Counting numbers, other kinds of number, by actual counting and the use of aids.

6. Number bases - 2, 3, 4, 7, 10, 12, using as illustrations the computer, market days, the week, the denary number system and the calendar year.

7. The operations of addition, subtraction, multiplication and division with reservation as to the need for teaching long multiplication and division skill.

8. The basic laws of arithmetic (and algebra) - commutative, associative and distributive.

9. Measures of money, time, weight, length, area, volume, by actual measurements and handling of objects in and around the classroom.

10. Other measures, e.g. temperature, rainfall, pressure, density, leading to statistical data.

11. Elementary graphs and charts.

12. Number patterns, tabular and graphic representations.

13. Transformations of translation, rotation, reflection, enlargement.

14. Direction, angles, simple plane figures, shadows and sun-dials.

15. Shapes of solid and plane figures, simple calculation of areas and volumes.

16. Measures in the mother tongue units and the metric system and the equivalents to one another.

17. Practical applications and illustrations, including games which involve mathematical ideas and patterns, such as ludo, snakes and ladders, dominoes, draughts, chess and local games.

Selected Teachers' Books

1. Association of Teachers of Mathematics: Notes on Mathematics in Primary Schools, London, Cambridge University Press, 1967.

2. A.T.C.D.E.: Primary Mathematics for Schools and Training Colleges, London, A.T.C.D.E., 1963.

3. Biggs, E.E. and Maclean, J.R.: Freedom to Learn: Ontario, Addison-Wesley (Canada) Ltd., 1969.

4. British Council: New Approaches to Teaching Mathematics (a book list), London, British Council, 1970.

5. Bryan Thwaites: On Teaching Mathematics, London, Pergamon Press, 1961, 116p.

6. Cockcroft, W.H.: Your Child and Mathematics London, W. & R. Chambers and John Murray, 1968, 28p.

7. Dienes, Z.P.: Mathematics in the Primary School, London, Macmillan, 1964.

8. H.M.S.O.: Children and their Primary Schools, London, H.M.S.O. Volume 1, 1967, 555p.

9. H.M.S.O.: Mathematics in Primary Schools, London, H.M.S.O., 1966, 165p.

10. Jay D. Weaver and Charles T. Wolf: Modern Mathematics for Elementary Teachers, Scranton (Pennsylvania), International Textbook Company, 1968, 274p.

11. Kennedy, J.: Understanding Sets, London, Thomas Nelson and Sons Ltd., 1967.

12. Lackie, L.: Understanding Shapes and Solids, London, Thomas Nelson and Sons Ltd., 1967.

13. Land, F.W. (Ed.): New Approaches to Mathematics Teaching, London, Macmillan, 1963.

14. Lee, D.M.: Background to Mathematical Development, London, Oldbourne, 1962.

15. Lucienne,Felix: Modern Mathematics and the Teacher, Cambridge, Cambridge University Press, 1966, 128p.

16. Meredydd,G. Hughes: Modernising School Mathematics, London, G. Bell & Sons Ltd., 1962, 43p.

17. Polya, G.: How to Solve It, New York, Doubleday, 1957, 253p.

18. Rodda, G.M.: Understanding Number, London, Thomas Nelson and Sons Ltd., 1967.

19. Rodda, G.W.: Understanding Graphs and Statistics, London, Thomas Nelson and Sons Ltd., 1968.

20. Schools Council: Mathematics in Primary Schools, London, H.M.S.O., 1966.

21. Scopes, P.G.: Mathematics for Primary School Teachers, London, Longmans, 1969, 120p.

Periodicals

1. Association of Teachers of Mathematics: Mathematics Teaching (the quarterly bulletin of the Association of Teachers of Mathematics), subscriptions to Hon. Treasurer Ian Harris, 122 North Road, Dartford, Kent, England.

2. The Mathematical Association: The Mathematical Gazette (the journal of the Mathematical Association), London, G. Bell and Sons Ltd., Portugal Street, London, W.C.1.

Notes:

1. Both periodicals carry a wide range of reviews and advertisements of mathematics books.

2. Back numbers contain illuminating articles on primary school mathematics.

Selected Pupils' Textbooks

The choice of pupils' books depends on what is available locally or can be obtained with minimum difficulty and on the extent to which the teacher depends on pupils' books. The ideal books are those specially written with the children and their environment in view. Some of them are in mother tongues other than English. The next best are those books which require little adaptation on account of their general appeal to children and their interests.