4. MATHEMATICS IN SECONDARY SCHOOLS[1]

A consideration of mathematics in secondary schools might begin with three fundamental types of questions. The first type revolves around what to teach and includes such questions as, "In an age of calculating machines need children know multiplication tables? Should we teach manipulation in algebra? If so, how much? Are logarithms obsolete? Should we teach trigonometry at all? If so, why? What parts of recent mathematics ought to come into the school syllabus? What should go out to make room for new topics? More fundamental, what considerations should determine our choice of syllabus?" The second type deals with the equally and perhaps even more important problem of how we teach. "Should mathematics fuse with science and the study of the environment, or should it be taught as abstractly as possible? Should it be formal or informal, rigorous or intuitive? In what proportions should discovery and 'telling' be mixed?" The third type concentrates on the objectives of mathematical education in secondary schools and asks how they can be achieved. This type asks why we teach what we teach in the way we teach it? How do we achieve our objectives?

The answers to these questions would vary from country to country and might vary from place to place in the same country, depending on the kind and quality of the teachers, the children and their interests, present and future needs, the aims and aspirations of the nation, the financial resources of the country and the general and special attitudes of the people to mathematical and other forms of education. Mathematical education in secondary schools should be based on the two strands of objectives which are already outlined[2] and which should be liberally interpreted to cover the various utilitarian aspects and the higher ideals of education. A minimum programme should achieve the following objectives:

(1) The pupil should enjoy mathematics and not be afraid of thinking about it.

(2) He should be able to grasp mathematical results informally, pictorially or in terms of a concrete situation.

(3) He should have worked with mathematics in connection with simple scientific laws and with the environment generally.

(4) In this way he should have acquired as much knowledge and understanding of arithmetic, algebra, geometry (and perhaps other mathematical subjects) as possible.

In many countries, secondary education consists of five years of broadly based education and two years of narrow but more specialised forms of education, usually referred to as Sixth Form. The first three years are devoted to an even broader educational base. In most countries, the Sixth Form is the preparatory class for University work. Pupils who select mathematics for their specialisation are in the range of the able and gifted children and their teachers are usually in the run of the better qualified teachers of mathematics. The mathematical education of the bigger group, that is Classes (Forms) One to Five needs special attention so that the

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[1]Adapted partly from the lead paper by Professor W.W. Sawyer.

[2]See p.3.

approach already started in the primary school may continue and that adequate mathematics may be provided in pursuance of the objectives of mathematical education at that level.

In planning a secondary school curriculum in mathematics, knowledge of the primary school background as to content and approach is important. The selection entrance examinations (where they are held) should concern themselves not only with the acquisition of skills but also with the development of understanding and attitudes. It is desirable that teachers should have considerable freedom in deciding syllabus content, although it would be unrealistic to think that all teachers are yet ready for this responsibility. The setting up of Syllabus Committees in countries where they do not exist should be encouraged. On these committees teachers should play a dominant role, but personnel from industry and Government should be included. In cases where external examinations are set, the examining bodies should be guided by local requests.

A school curriculum should take cognisance of the school organisation and facilities. Timetables should be arranged to permit 'setting' in mathematics. Under this arrangement a group of classes in a particular year have mathematics simultaneously, the students being assigned to classes according to ability in mathematics. This arrangement allows a student to transfer at any time during the year to a class which accommodates his ability. Classroom furniture should be chosen so as to facilitate group working. Flat-top tables are preferred to sloping ones. Where possible mathematics teachers should have individual classrooms to which classes may go in turn. Failing this, there should be a mathematics room or a laboratory suitably equipped. This moderate requirement may be beyond the resources of some schools but it should be kept in view as a minimum requirement.

All children in the first three years of their secondary education should pursue courses with a common core. Those children who are more able in mathematics should be given the opportunity to study topics in greater depth and to consider additional topics. At the other end of this ability range, topics should be treated in a more informal manner, but at the same time such children should not be deprived of an enrichment programme. The following principles for the design of a mathematics course are recommended:

(i) The establishment of a natural link with primary school teaching which should make the transition from primary to secondary school as smooth as possible.

(ii) In some situations a practical approach involving student activity would be a useful teaching technique to use. Teachers of mathematics in secondary schools should be familiar with and take an interest in primary school methods and encourage the continuance of these where appropriate.

(iii) In secondary schools there should be a shift of emphasis to systematising and formalising, as well as expanding the concepts and experience of the primary school.

(iv) To make the course relevant to the community and its mathematical needs as far as they can be identified.

(v) The school curriculum should be integrated so that the contribution of a subject to each of its related subjects can be used to the best advantage e.g. the mathematical concepts necessary to physics should be co-ordinated with the physics programme and, on the other hand, aspects of physics which could serve to illuminate the need for mathematical concepts should be co-ordinated with the mathematics programme.

(vi) Proof or some other form of justification should be given for all formulae, algorithms and theorems considered.

(vii) In cases where the child's formal education is to terminate after 3 years, he must leave school not only with an acceptable degree of numeracy (i.e. insight into mathematical thinking in general and into all basic arithmetical techniques) but also with that flexibility of mind which would enable him to apply himself to problems he will meet as a member of his community.

In considering the needs of pupils in their first three years of secondary education no attempt is made to lay down a syllabus or to suggest the order in which topics should be taught. The depth of treatment of these items, and the way in which they would be introduced would vary from country to country and indeed from class to class or even pupil to pupil. For example, although all pupils would be expected to be able to solve linear equations, quadratic equations would be included only when there was a clear need. The following topics are suggested as those which should be encountered by all students:

<u>Set</u>

The idea of a set, set language and notation; union, intersection, complement, empty (null) set, universal set, subset; illustrations by Venn diagrams; the basic laws of set.

<u>Number</u>

Number line, extension to negative numbers, rationals, irrationals, and reals.
Place value, scientific notation (e.g. 2.38×10^{-6}).
Computational aids (e.g. the slide rule).
Percentage, ratio, approximation and error estimation.

<u>Algebra</u>

Variable, functions.
Algebraic expressions and operations.
Relations, equivalence relations.
Inequalities, identities, formulae.
Equations, solution of equations.
Graphical representation.

<u>Geometry</u>

Area and volume, the theorem of Pythagoras.
Similarity, angle measure and angle properties.
Elementary trigonometry.
Plans and elevations.
Co-ordinate geometry.
Transformations of translation, rotation, reflection, enlargement.

Identity elements and inverse elements should be studied in geometrical as well as in numerical and algebraic contexts.

Statistics and Probability

It is suggested that consideration of probability should spring from the pupil's own experiments, and that the statistics studied should be confined to representing data by means of graphs or tabulation, the interpretation of data exhibited in these forms, and the use of the measures mean, mode and median.

Terms such as equivalence relation are not expected to be used by pupils, or necessarily to be defined explicitly, but nevertheless the ideas should permeate the teaching. For example, children should realise the similarity between 'is equal to', 'is parallel to' and 'was born in the same year as'.

It must be stressed that this skeleton plan must be supported by a full enrichment programme appropriate to the pupils concerned. For example, matrices might be studied not only for their own sake but also to increase the pupil's understanding of such items as operations, equations, inverse elements and co-ordinate geometry; the study of transformation or vector geometry would, amongst other things, greatly increase a child's experience of spatial relations. However, the choice of such topics should be left to individual countries or teachers.

When considering the syllabus for the fourth and fifth years, it is desirable to bear in mind the fact that for many students this would mark the end of their formal education in mathematics. For such students topics with social and economic relevance should be given particular consideration. The following topics are thought to be appropriate:

(a) Linear programming.

(b) Statistics up to sample theory and significance (including projects of an experimental nature).

(c) Co-ordinate geometry and vectors.

(d) Operatives in systems other than the real numbers e.g. matrices, geometrical transformations.

(e) Computing, flow charts, programming in some subsets of high level language, Computer appreciation.

(f) Social arithmetic e.g. income-tax, hire-purchase, etc.

The choice of topics at the Sixth Form level should be sufficiently wide to satisfy the needs of

(i) those whose formal education in mathematics will terminate at the end of this stage;

(ii) future social and biological scientists;

(iii) future physical scientists and engineers;

(iv) future specialists in mathematics.

Some of the principles which should guide the choice of content are:

(i) the need to systematise parts of mathematics, for instance, co-ordinate geometry, properties of polynomials, elementary number theory;

(ii) the need for refining the ideas of proof and the use of axiomatic processes where possible. For example, elementary abstract algebra can be used to justify rigorously results presented informally in the earlier grades. On the other hand, the introduction to the calculus should be based on an intuitive approach to limits;

(iii) the need to make mathematical models of physical situations: i.e. abstracting from a physical situation by symbolising quantities and relations and arriving at mathematical conclusions in the model which can be interpreted in terms of the physical situation. Such illustrations are to be found in mechanics and probability;

(iv) the need to develop facility in the use of mathematical methods.

Patterns of collaboration with industry and technology depend on each country, its resources and circumstances. Collaboration might take the form of getting schools, industry and technology to define aims and scope of the mathematics curriculum for the country, or of representatives of industry and technology participating in teaching in the schools and in the process of examinations. There are examples of national forums for curriculum development.

Modern mathematics in the secondary school should reflect in approach and content the objectives of a good mathematical education, bearing in mind the needs of the country and the power of mathematics not only as an intellectual exercise but also as an instrument for the general education of the person.

Suggestions for Action

In many countries, a secondary school mathematics teacher is a specialist teacher. He is usually the holder of a degree in mathematics or in a combination of two or three subjects of which mathematics is one. He may in addition have a teaching qualification. If he is not a graduate, he would have had two or three years of teacher training and further mathematical education beyond the level represented by the ordinary level of the General Certificate of Education in mathematics. In countries where teachers with lower qualifications teach mathematics in a secondary school, the situation is regarded as an emergency requiring urgent attention. The Sixth Form mathematics teacher usually holds an honours degree in mathematics or its equivalent.

It is essential for the teacher to bring himself up-to-date in the aims, content and method of teaching modern mathematics,[1] through reading,

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[1] A detailed account of the terms "Traditional", "Modern" and "New" Mathematics is contained in the Unesco Mathematics Project for the Arab States, published by Unesco in January 1969.

attentance at conferences, seminars and in-service courses. A practical step for the teacher is to subscribe to a mathematics professional journal like the Mathematical Gazette which carries contemporary articles, reviews and advertisements of mathematics books. The secondary school teacher should acquaint himself with the primary school mathematics curriculum, its objectives, content and approach.

The following suggestions are made with the above basic assumption of the teacher's competence and continued up-dating:

(1) Links between primary and secondary schools should be established to ensure that transition is as smooth as possible.

(2) Teachers should be given considerable freedom in deciding syllabus content. It is often desirable to set up a syllabus committee on which teachers play a major rôle and both industry and Government are represented.

(3) Secondary courses should be relevant to the various mathematical needs of the community in which they operate.

(4) In the first three years of secondary schooling, mathematics courses should have a common core; thereafter they should be varied to suit the needs of particular groups.

(5) Mathematics and other subjects of the curriculum should be inter-related so that the contribution of each subject can be used to the best advantage.

(6) Timetables should be planned to allow simultaneous teaching of different ability groups within a particular year.

(7) To facilitate group working and practical classroom activities, suitable furniture should be chosen.

(8) It is essential to provide a mathematics room and, where possible, a suitably equipped mathematics laboratory.

(9) In most educational systems of the English-speaking countries, teachers have freedom to try new methods and influence the curriculum and the syllabus. Advantage of that freedom should be taken to to try group work and individualised teaching, identify the able and the weak pupils and help them develop according to their ability and needs and to select those topics which assist understanding and promote skill.

(10) A number of examining bodies are disposed to co-operate with teachers and base their examinations on representative curricula. They welcome criticisms of the examinations and their relevance to the school curricula. That freedom should be exploited to ensure that examinations are based on the curricula and not the other way round.

(11) Many school certificate mathematics examinations consist of two sections, one on the core topics and the other on a wide range of options. Teachers should use the advantage of the pattern to select the topics which best fulfil their objectives of mathematical education.

Selected Teachers' Books

1. Austin, J.L.: The Foundations of Arithmetic, Oxford, Basil Blackwell, 1950.

2. Board of Education: Curriculum and Examinations in Secondary Schools, London, H.M.S.O., 1946.

3. Bruce, George: Secondary School Examinations Facts and Commentary, London, Pergamon, 1969.

4. French, P.: Number Systems, London, The House of Grant Ltd., 1964.

5. Hardy, G.H.: A Mathematician's Apology, Cambridge, Cambridge University Press, 1940.

6. Incorporated Association of Assistant Masters in Secondary Schools: The Teaching of Mathematics, Cambridge, Cambridge University Press, 1960.

7. Land, F. W. (ed.): New Approaches to Mathematics Teaching, London, Macmillan, 1963.

8. Meredydd,G. Hughes: Modernising School Mathematics, London, G. Bell & Sons Ltd., 1962.

9. Ministry of Education: Teaching Mathematics in Secondary Schools, London, H.M.S.O. 1958.

10. Ministry of Education: The Road to the Sixth Form, London, H.M.S.O., 1951.

11. Ministry of Education: 15 to 18, London, H.M.S.O. 1959.

12. Nunn, T.P.: The Teaching of Algebra, London, Longmans, 1941.

13. Peel, E.A.: The Pupil's Thinking, London, Oldbourne, 1967.

14. Pedoe, Dan: The Gentle Art of Mathematics, London, Pelican,1965.

15. Polya, G.: How to Solve It, New York, Doubleday, 1957.

16. Reichmann, W.J.: The Fascination of Numbers, London, University Paperbacks, Methuen, 1965.

17. Russell, Bertrand: Introduction to Mathematical Philosophy, London, George Allen and Unwin Ltd., 1950.

18. Sawyer, W.W.: A Concrete Approach to Abstract Algebra,London, W.H. Freeman and Co. 1959.

19. Sawyer, W.W.: Prelude to Mathematics, Harmondsworth, Penguin, 1955.

20. Sawyer, W.W.: Mathematician's Delight, Harmondsworth, Penguin, 1959.

21. Sawyer, W.W.: Vision in Elementary Mathematics, Harmondsworth, Penguin, 1964.

22. Sawyer, W.W.: A Path to Modern Mathematics, Harmondsworth, Penguin, 1966.

23. Secondary School Examinations Council: The Certificate of Secondary Education - some suggestions for teachers and examiners. London, H.M.S.O., 1963.

24. Secondary School Examinations Council: The Certificate of Secondary Education - An Introduction to some Techniques of Examining, London, H.M.S.O., 1964.

25. The Mathematical Association: The Mathematical Gazette (the journal of the Mathematical Association), London, G. Bell and Sons, Ltd.

26. The Mathematical Association: A Second Report on the Teaching of Arithmetic in Schools, London, G. Bell and Sons Ltd., 1964.

27. The Mathematical Association: The Teaching of Arithmetic in Schools, London, G. Bell and Sons Ltd., 1946.

28. The Cambridge University Mathematics Society: Eureka (the Journal of the Archimedeans), Eureka c/o The Arts School, Benet Street, Cambridge, England.

29. The National Foundation for Educational Research in England and Wales: Educational Research, London, Newnes Educational Publishing Co.

30. The Mathematical Association: A Second Report on the Teaching of Mechanics in Schools, London, G. Bell and Sons Ltd., 1965.

31. The Mathematical Association: Transfer from Primary to Secondary Schools, London, G. Bell and Sons Ltd., 1964.

32. Thwaites, Bryan: On Teaching Mathematics, London, Pergamon Press; 1961.

33. UNESCO (Education Clearing House): The Teaching of Mathematics by G. Mialaret, Paris, Unesco, 1959.

34. University of Southampton: Aspects of Modern Mathematics, Southampton, The University, 1963.

35. Vernon, Philip E.: The Certificate of Secondary Education - An Introduction to Objective-Type Examinations (Examinations Bulletin No. 4), London, H.M.S.O. 1964.

36. Wertheimer, M.: Productive Thinking, New York, Harper, 1945.

Selected Pupils' Textbooks

There is a wide range of secondary school mathematics books written in the developed countries and distributed widely in developed and developing countries. The practice is based partly on the assumption that mathematics is ultimately culture-free. However, there is a welcome trend of developing mathematics curricula and syllabuses on a country-basis; authors in the future will write books specifically aimed at each country or region. In this trend, two predominant sources are the School Mathematics Project (S.M.P.) of the United Kingdom and the School Mathematics Study Group (S.M.S.G.) of the United States of America, supported by the Illinois Experimental Programme. Books by the Scottish Mathematics Group (S.M.G.) have appeared in West Africa as have some single-author books.

The School Mathematics Project books are used in many African countries, East and West. They are being adapted and re-written to suit local regions. Thus there are in East Africa (with Makerere as the base) the School Mathematics Project for East Africa and, in Ghana, the Joint Schools Project (J.S.P.) which produce books based on the S.M.P. material.

The ideas of the S.M.S.G. and the Illinois Experimental Programme came through the Educational Services Incorporated (E.S.I.) and now the Education Development Center (E.D.C) which organised the African Mathematics Programme. The books produced under this programme are known as Entebbe Mathematics, after Entebbe, Uganda, where most of the workshops for preparing the material were held. In the curriculum development in the developing countries, there will emerge new material incorporating the best from both sources.

A number of publishers have modern mathematics books, which are worth 'inspecting'. Whatever textbooks are adopted, secondary school pupils should be encouraged to read a wide range of mathematics books, including biographies, histories, pastimes. As a minimum reading, Bell, Hardy and Sawyer are recommended. Here is a short general reading list.:

1. Austin, J.L.: The Foundations of Arithmetic, Oxford, Basil Blackwell, 1950.

2. Bell, E.T.: Men of Mathematics, Vols. 1 & 2, Harmondsworth, Penguin, 1937.

3. Hardy, G.H.: A Mathematician's Apology, Cambridge, Cambridge University Press, 1940.

4. Kennedy, J.: Understanding Sets, London, Thomas Nelson and Sons Ltd., 1967.

5. Lackie, L.: Understanding Shapes and Solids, London, Thomas Nelson and Sons Ltd., 1967.

6. Pedoe, Dan.: The Gentle Art of Mathematics, London, Pelican, 1965.

7. Polya, G.: How to Solve It, New York, Doubleday, 1957.

8. Reichmann, W.J. : The Fascination of Numbers, London, University Paperbacks, Methuen, 1965.

9. Rodda, G.W.: Understanding Number, London, Thomas Nelson and Sons Ltd., 1967

10. Rodda, G.W.: Understanding Graphs and Statistics, London, Thomas Nelson and Sons Ltd., 1968.

11. Russell, Bertrand: Introduction to Mathematical Philosophy, London, George Allen and Unwin Ltd., 1950.

12. Sawyer, W.W. A Path to Modern Mathematics, Harmondsworth, Penguin, 1966.

13. Sawyer, W.W.: Mathematician's Delight, Harmondsworth, Penguin, 1959.

14. Sawyer, W. W.: Prelude to Mathematics, Harmondsworth, Penguin, 1955.

15. Sawyer, W.W.: Vision in Elementary Mathematics, Harmondsworth, Penguin, 1964.

The following book lists are useful in selecting books for a School Library.

1. The British Council: New Approaches to Teaching Mathematics, London, 1969.

2. The Mathematical Association: School Library Mathematics List, London, G. Bell and Sons Ltd., 1966.

3. University of Southampton: Aspects of Modern Mathematics, Southampton, the University, 1963.

The School Mathematics Project books are published by Cambridge University Press, Cambridge and the Entebbe Mathematics books by the Education Development Center, Newton, Massachusetts, U.S.A.