7. RESOURCES FOR LEARNING MATHEMATICS[1]

The Environment

Resources for learning mathematics are part of our total environment, natural and man-made. The "mathematical content" of our natural environment is much the same for all of us, wherever we happen to be located. The shape of the sun; the apparently changing shape of the moon; daily and seasonal patterns of shadows; regularity and symmetry in crystals, leaves, flowers, and seeds; experiences with water, such as reflections, waves and ripples; - these are freely available to all. We know that many of these things have had a significant effect on the historical development of mathematics, and it is not unreasonable to believe that we should be able to exploit them as resources for the learning of mathematics.

The man-made environment includes all aspects of our total environment which are, directly or indirectly, the creation of man. This includes not only those specific products of our manufacturing technology but also such other man-made creations as our homes; our cities; our social, political and economic systems (including our use of money); our many devices for the transportation of goods, information and ideas; and the many gadgets which flow from the application of our advancing science and technology. The natural environment has played a well-known and significant role in the historical development of mathematics, but the man-made aspects of our environment (and our need to understand and further develop them) have now assumed a far greater significance in relation to current mathematical developments and current curriculum developments.

While we seem to be moving slowly in the direction of greater uniformity in the technological aspects of our societies, there are still very great differences from one place to another. Whether or not this implies the need for differences in mathematics curriculum, there is little doubt that these environmental differences deserve serious consideration in relation to the use of resources for implementing whatever curriculum is used.

The profound influence of our cultural environment on our personal development is, of course, well known to psychologists and anthropologists but we sometimes tend to overlook it in relation to the learning of such a universal and apparently culture-free subject as mathematics. A belief shared by many is that not enough attention has been paid to cultural differences, both in the design of curricula and in the use of various resources for learning mathematics. It might be hopelessly visionary to contemplate a time when the design of mathematics curricula and the use of learning resources will be fully adapted to the needs and potentialities of each individual, but it is not quite so unrealistic to suggest that more attention (especially in the use of resources) should be paid to some of the more obvious cultural differences. This implies, for example, that books designed for use in one society should be "culturally translated", as well as verbally translated, in order to adapt them for the needs of another. Of course, many other dichotomies are possible in addition to such well known ones as rural-urban, affluent-poor, and culturally "normal"-culturally deprived. One which has received a great deal of attention in recent years is related to the relative state of technological advancement, classifying countries rather roughly as "developed" and "developing".

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[1] Adapted largely from the lead paper by Professor A.L. Blakers.

A study[1] by two teachers from one of the most technologically developed countries (the United States of America) in teaching elementary mathematics to children from the Kpelle tribal group in Liberia suggests most strongly that the teacher should understand the culture in which he is working, and how this affects not only the suitability of what is taught (the curriculum) but also the choice of a teaching strategy (including the use of learning resources) which is likely to prove most effective. Another but independent study[2] of social mathematics in a Yoruba society emphasises the need to understand the cultural background and practices of the children in order to assist them effectively in forming mathematical concepts. The Kpelle and the Yoruba tribes are not isolated cases of environmental factors in learning mathematics. The choice of curriculum and the use of available learning resources should take into account the nature, and the changing nature, of the cultural background of the children, and of the future environment in which they will live. If we can discover how to do this successfully, then it will be possible to exploit the environment itself much more than we do as a resource for learning mathematics.

The Teacher

In a list of resources for learning mathematics, the teacher is topmost. The best-conceived curriculum would founder if due attention were not paid to the crucial role of the teacher and to the improvement of teachers themselves, as well as the improvement of other resources such as textbooks and teachers' guides. It is the teacher who is still the main channel for the communication of mathematical ideas to the child, no matter what curriculum is being studied. This statement is as true for classes using the many fine new textbooks that have recently appeared, as it is for classes using older books or having no textbooks at all. And, as far as one can tell at present, the role of the teacher is likely to be just as significant (even if different in detail) in conjunction with such "automated" teaching methods as programmed learning and computer assisted learning. In many ways, the teacher is the critical classroom resource, who sets a limit to the effective use of every other resource - textbooks, films and film strips, programmed materials, attribute blocks, calculators, overhead projectors, number rods and other analogy devices, mathematical games and puzzles, and the many other "teaching aids" which are appearing in our classrooms.

Books

Books are an essential resource for learning mathematics. Books have been with us for a very long time, and their role as carriers of various aspects of our human culture is well known. The two great storehouses for the accumulation and transmission of mathematical knowledge have been people and books. A little reflection shows the extent to which our personal development - in mathematics as in other areas - has been assisted by our use of books.

In the production of books, one interesting development of the last decade (closely associated with curriculum reform) has been the growing tendency for the use of fairly large groups of writers, rather than the more

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[1] Gay, J. and Cole, M.: The New Mathematics and an Old Culture, New York, Holt, Rinehart and Winston, 1967.

[2] Taiwo, C.O.: Social Mathematics in a Yoruba Society (Unpublished Thesis accepted by the University of London).

traditional authorship by one, or by a small number of writers. This group writing is seen in the work of such bodies as the School Mathematics Study Group, the School Mathematics Project, the African (Entebbe) Mathematics Project, the Joint Schools Project of the Mathematical Association of Ghana, the School Mathematics Project of East Africa and many others. There is no doubt that such group writing can result in a very critical selection and appraisal of material, although it sometimes runs into problems of style. The potentialities of group writing have not been fully realised in most of the recent projects, due to rather severe restrictions of time and money. But, given an adequate supply of both, it seems safe to conjecture that many of the best textbooks of the future will be produced by the co-operative efforts of mathematicians, teachers, learning theorists, and publishers, assisted by feedbacks from the pupils on whom the texts are evaluated.

There have also been significant changes in recent years in the art and technique of book production. Examples are the attractive use of art work and colour in the books of the Nuffield Mathematics Project,[1] the Houghton Mifflin Modern Mathematics series,[2] the New Oxford Junior Mathematics[3] and Freedom to Learn:[4] these books are a far cry from the generally drab textbooks of a generation ago. Another innovation is in the use of transparent overlays. Examples are the School Mathematics Project and the Houghton Mifflin Modern Mathematics series.

Of considerable importance are the increasing number and the widening range of mathematics books which could assist teachers and pupils in learning mathematics. There are the general books, textbooks, reference books, monographs, books on mathematical games, recreations and puzzles, biographical and historical books, journals and bulletins. A number of them, published by one mathematical association or another, are sold at minimal prices. School Libraries and individual teachers may make good collections at a moderate cost. There are useful bibliographies and reviews in journals and bulletins, which supply helpful information for collectors.

In many developing countries, there are few school mathematics textbooks which are written with the background, interest and needs of the children of the respective countries in mind. Some of the so-called adapted textbooks are little more than the foreign books with changed or modified titles. Some of the textbooks in use in some of these countries are old and out-of-date. In countries where a local language other than English is the medium of instruction in the early years at school, mathematics textbooks in the language are extremely few and usually they do not exist. There is therefore need for publishers and authors to co-operate in supplying suitable textbooks in developing countries. There is a place for books on modern methods of teaching mathematics as well as modern content and for background books which would enrich the mathematical education of pupils and teachers.

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[1] Nuffield Mathematics Project: series published by John Murray Ltd., 50 Albemarle Street, London, W.1.

[2] Duncan, Capps, Dolciani, Quast, Zweng: Modern School Mathematics, Structure and Use K-6, Boston, Mass., Houghton Mifflin Company.

[3] Williams, E.M. and James, E.J.: New Oxford Junior Mathematics, London, Oxford University Press, 1971.

[4] Biggs, E.E. and Maclean, J.R.: Freedom to Learn, Don Mills, Ontario, Addison-Wesley (Canada) Ltd., 1969.

Audio-Visual Aids - Films, Film Strips, Television, Radio

Films, film strips, television and radio are resources which involve capital expenditure quite out of the reach of many schools. When they are available they are useful in supplementing the effort of the teacher, compensating for the shortage of qualified teachers and strengthening students and young teachers in their teaching methods.

A slide projector is considered to be of very limited use in the teaching of mathematics and it is being superseded by the overhead projector. The advantages of the overhead projector are

(i) That not only could straight pictures be put on, but a sequence of transparencies could be used to build up a composite picture as a lesson proceeds.

(ii) The structure of such overhead projectors makes it possible for shadows of solid objects to be cast on the screen. In the teaching of motion geometry this would prove a very valuable visual aid.

Television can be used for the following purposes:

(i) To explore regions of mathematics which lend themselves (or could lend themselves) to visual interpretation, especially of a type that teachers cannot easily cope with for one reason or another. The means to this end are very varied but usually involve film animation, electronic wizardry, models and animated captions.

(ii) To explore fresh areas of mathematics with which most pupils and many teachers might not be familiar. This often means re-interpreting rather difficult books and inviting university and college lecturers to the studio. Occasionally it involves extensive actuality filming (e.g. of computers in operation).

(iii) To explore the mathematics all around by drawing on examples from everyday life and by showing applications in science, technology, sociology, etc. Apart from the techniques already mentioned, photographs are often of value here.

(iv) To use the medium to "pipe" all kinds of mathematics to schools which are short of qualified staff. This can allow more effective use of the time of qualified teachers either by themselves or in some form of team-teaching.

(v) To encourage and advance the work of slow learning pupils. Programmes produced by the B.B.C. for slow readers using the full gamut of visual techniques have been particularly successful. One of the most surprisingly successful devices used for encouraging children's work at all levels of ability has been that of showing films of other children working.

(vi) To inform teachers of developments in the subject matter and teaching of the subject using all the techniques mentioned above.

(vii) To act as a source of secondary material in the form of books, films, etc.

Where the facilities exist, video tape may be used in teacher training institutions to enable the students view their own lessons and criticise themselves. It may also be used to record and supplement transmitted programmes.

The use of radio in the teaching of mathematics is of limited value. Mathematics has its language, notation and symbols and lends itself more to visual than to aural presentation. It may however be used with correspondence courses with a view to reinforcing such courses.

One of the advantages of using television and radio is that the wider audience of parents and other interested parties could be reached as well as the pupils and teachers to whom the programme is really directed. This is a great advantage when considering any form of educational change whether of mathematical content or of approach.

Programmed Learning

Programmed learning has made little impact in the teaching of mathematics in developing countries. In the countries where it is being used, it appears not to have lived up to some of the claims made for it. A great deal of experimental work on programmed learning is in progress and may lead to establishing it as a useful resource in supplementing the work of the teacher, but certainly not in replacing him.

Low Cost Teaching Aids and Demonstration Equipment

In the production of low cost teaching aids, emphasis should be on the use of local materials and the low cost. Guidance as to what to make and how to make and use it may be found in the mathematical journals and bulletins of the developed countries. Numbers 18 and 24 of Mathematics Teaching, Journal of Teachers of Mathematics in the United Kingdom, are devoted to structured and other aids.

An important consideration in buying demonstration equipment and gadgets is the cost of buying and maintaining them. In developing countries, this could be prohibitive. Many a time, maintenance service is not adequate or available. A number of these gadgets are made without sufficient precaution against the weathering and deteriorating effect of the tropics. Efforts are being made by Ministries of Education in some countries to set up production centres where suitable aids and equipment are made from local materials and with emphasis on relevance to the local circumstances.

Priorities of Resources

By way of summary, it is desirable to set the priorities in the value of the resources for learning mathematics at the primary and the secondary school levels. In the primary school, the priorities should be

(i) Qualified teachers
(ii) Materials arising from the equipment
(iii) Books (text, reading, reference)

and in the secondary school

(i) Qualified teachers
(ii) Books (various categories comprising text, background, reference, recreations, games, applications)
(iii) Physical equipment to assist investigation and discovery.

Suggestions for Action

1. The teacher is the most important of all the many and varied resources for learning mathematics. He should therefore be well qualified by his initial and continued education in mathematics and the methods of learning and teaching mathematics. He should know the environment of the child and how to present the environment at the different stages of the child's growth and development.

2. The teacher should be able to select the resources that bring excitement, interest and insight into the learning of mathematics by children. The selection should start with the child, things about his person and dress, in the classroom, the school yard, the home, the farm and the street. The child should become a partner in identifying and collecting suitable resources for counting, matching, measuring, weighing and comparing. Examples are chairs, tables, desks, books, pencils, seeds, shells, pebbles and counters, tins, containers of various shapes and sizes, measuring rods, tapes and rulers, string and weights, water and sand. With a variety of commonplace resources, children learn mathematics, not only at school but in the home, the farm and the playground.

3. The value of the resources for learning mathematics is the use that is made of them. The teacher must therefore be familiar with the resources, their use and their contribution to the learning of mathematics. He should then use the resources appropriate to his lesson and suited to the needs of the children.

4. An important resource is the blackboard (black and smooth-surfaced) and a generous supply of white and coloured chalks. These as well as graph boards, geo-boards and mathematical sets for use with the blackboards should be available in the school and used freely.

5. Books are valuable resources for learning mathematics. The teacher himself should cultivate a taste for reading mathematics and inspire his pupils to do the same. The books should range over textbooks, reference books, general books, books on games, recreations, puzzles, applications to other subjects such as science, technology and commerce. To promote education in reading mathematics, the school library should contain a good number and wide range of mathematics books. The teacher should also have a collection, which should include at least one journal or bulletin taken regularly.

6. Textbooks should have relevance to the pupils' background and experience and should cover a wide range of children's activities. This is the more desirable in primary school textbooks, which are an introduction to the reading of mathematics. In addition to the textbooks used by the pupils, the library should have a number of other good textbooks so that the teacher may be aware of other approaches and the pupils of the activities of children in other lands. An example of a good primary school series is the New Oxford Junior Mathematics produced in colours and containing a wide variety of children's activities.

7. Commercially produced structural materials such as Cuisenaire, Dienes', Stern, Structa, Avon, Colour Factor, Unifix and Venture which are used in the primary school for abstracting concepts of

place value and basic number facts and laws are a rare feature of the primary school in developing countries on account of their cost. In the circumstances, teachers in developing countries must concentrate on improvising and making mathematical aids from local materials, with the object of assisting children to make their abstractions.

8. In the secondary school, the workshop, the farm, the play fields, the laboratories and in fact the whole school premises are resources for learning mathematics. It is for the teacher to draw on these resources for his lessons. Examples are geometrical models (made in the workshop), plans, statistics leading to graphs, probability, sets, etc.

9. Audio-visual aids such as television, radio, films, film strips, slides, video tapes, overhead projectors are resources for learning mathematics and other subjects of the school curriculum. The use of these aids is limited by the availability of the facilities (television and radio) and the cost and maintenance of projectors and films, film strips and slides. Where the facilities exist, the resources should be used to reinforce other resources and the teacher. In developed countries and the urban areas of developing countries, television and radio are used in education; films, film strips and slides and even projectors may be borrowed from libraries. In many schools, primary and secondary, this class of aids is yet a remote resource.

Audio-visual aids centres in the University Departments and Institutes of Education and Colleges of Education (Advanced Teachers Colleges) have these aids and give courses on their use and maintenance to students and teachers. Mathematics Teaching No. 24[1] and Freedom to Learn[2] contain assessments of the aids.

10. It is to the teacher we turn for popularising mathematics and finding ways and means of creating the atmosphere for learning mathematics. The organisation of school mathematics clubs, formulation of mathematical projects, teachers' associations, exchange visits, excursions to industrial works, and a number of other activities and ideas depend for their success on the teacher.

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[1] Association of Teachers of Mathematics: Mathematics Teaching No.24

[2] Biggs, E.E. and Maclean, J.R.: Freedom to Learn: Ontario, Addison-Wesley (Canada) Ltd., 1969.

Selected Reading List

1. Association of Teachers of Mathematics: Mathematics Teaching Nos. 16-20, 22, 24; 122 North Road, Dartford, Kent, England.

2. Biggs, E.E. and Maclean, J.R.: Freedom to Learn, Don Mills, Ontario, Addison-Wesley (Canada) Ltd., 1969.

3. Cundy, H.M. and Rollett, A.P.: Mathematical Models, London, Oxford University Press, 1961.

4. Davis, R. Mathematics: The Changing Curriculum, Washington, Association for Supervision and Curriculum Development, N.E.A., 1967.

5. Gay, J. and Cole, M.: The New Mathematics and An Old Culture, (A Study of Learning Among the Kpelle of Liberia). New York, Holt, Rinehart & Winston, 1967.

6. McLuhan, M.: Understanding Media, London, Routledge & Kegan Paul, 1964.

7. National Council of Teachers of Mathematics: Mathematics Teacher, 1201 Sixteenth Street, Washington 6, D.C.

8. Nuffield Mathematics Project: I do and I understand; Pictorial Representation; Shape and Size; Computation and Structure, etc. London, John Murray, 1967.

9. Thwaites, Bryan: On Teaching Mathematics, London, Pergamon Press, 1961.

10. UNESCO: New Trends in Mathematics Teaching, Volume 1, Paris, UNESCO, 1967.